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Landau Pole Effects and the Parameter Space of the Minimal Supergravity Model

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Abstract

It is shown that analyses at the electroweak scale can be significantly affected due to Landau pole effects in certain regions of the parameter space. This phenomenon arises due to a large magnification of errors of the input parameters m_t , α_G which have currently a 10 percent uncertainty in their determination. The influence of the Landau pole on the constraint that the scalar SUSY spectrum be free of tachyons is also investigated. It is found that this constraint is very strong and eliminates a large portion of the parameter space. Under the above constraint the trilinear soft SUSY breaking term at the electroweak scale is found to lie in a restricted domain.

1. Introduction

Currently among the models beyond the standard model, only those with supergravity [1] can accommodate the unification of the electroweak and strong coupling constants that is consistent with the LEP data [2], provide a natural mechanism to break supersymmetry via a hidden sector [3] and can achieve a radiative breaking of the electroweak symmetry breaking via renormalization group effects[4]. The minimal supergravity model (MSGM) is one of the most studied of such models [4]. The parameter space of the MSGM with the radiative symmetry breaking consists of only four parameters:

$$m_0, \quad m_{\tilde{g}}, \quad A_t, \quad \text{and} \quad \tan\beta \quad (1.1)$$

in addition to the top quark mass, m_t (which is indicated by CDF to be $m_t = 174 \pm 15$ GeV [5]) and the GUT parameters, M_G (the grand unification scale), and α_G (the gauge coupling constant at M_G). Here in equation (1), m_0 is the universal scalar mass at M_G , $m_{\tilde{g}}$ and A_t are the gluino mass and the trilinear scalar coupling at the electroweak breaking scale, M_{ew} (we choose it to be the Z-boson mass, $M_Z = 91.187$ GeV), and $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$ is the Higgs vacuum expectation value (VEV) ratio at M_{ew} , where $\langle H_2 \rangle$ and $\langle H_1 \rangle$ give masses to the up and down type quarks, respectively. The parameter space given by eq (1) has two branches, corresponding to the two signs of the supersymmetric Higgs mixing parameter, μ , since radiative symmetry breaking determines only the magnitude of this parameter.

Although the MSGM has only four free parameters, in addition to the 19 for the Standard Model[and $137 + 19$ for the minimal supersymmetric standard model (MSSM)], the parameter space is still quite large and one may put additional constraints on the model to further reduce the allowed range of parameters. Constraints that have been used in the past consist of proton stability for the SU(5) type model [6], the neutralino relic density constraint consistent with the current estimates of dark matter in the universe [7], the $b \rightarrow s\gamma$ decay constraint [8,9], as well as constraints on the lower bounds on the SUSY particle masses from CDF, D0 and LEP experiments. Thus for models with proton decay, such as the SU(5) type models, proton stability requires $m_0 > m_{\tilde{g}}$ and $\tan\beta \lesssim 10$. The assumption of R parity invariance implies that the lightest neutralino (\tilde{Z}_1) is the

lightest supersymmetric particle (LSP) for almost the entire parameter space. It does not decay due to R parity conservation, and hence may be regarded as the candidate for the cold dark matter in the universe. Recent analyses have shown that there exists a large domain in the parameter space which satisfies both the proton stability and the relic density bounds [7]. One may also impose additional constraints on the theory such as the experimental constraint of the $b \rightarrow s\gamma$ branching ratio. This decay is interesting as it is one of the few low-energy processes that is sensitive to physics beyond the standard model [10], and the first experimental measurement of this process has been made [11]. Recent analyses [8,9] show that there is a significant region of the parameter space where the branching ratio $B(b \rightarrow s\gamma)$ lies within the current experimental bound even when combined with the relic density and proton stability constraints.

In addition to the constraints discussed in the previous paragraph, the internal consistency of the theory also puts stringent constraints on the parameter space. For example, the color $SU(3)_C$ group should remain unbroken [12] including the one loop corrections to the effective potential [13] when the radiative symmetry breaking is imposed, all scalar particles must be non-tachyonic, and the allowed parameter space should be such that theory remains in the perturbative domain. It turns out that the restriction that the mass spectrum be free of tachyons imposes strong constraints on the parameter space of the model. An interesting aspect of minimal supergravity unification is its predictivity. There are 32 supersymmetric particles in the model whose masses are determined in terms of the 4 parameters of eq. (1.1). Thus the theory makes 28 predictions. However, there is an important issue regarding how accurately predictions of the mass spectra can be made. We shall show that the precision with which predictions are made can be significantly influenced by the proximity of the top quark to the fixed point in the top quark Yukawa coupling. It turns out that in certain parts of the parameter space, the proximity to the Landau pole can magnify errors in input data such as m_t and α_G (which are known to no more than 10 percent accuracy) by an order of magnitude. Thus in this region of the parameter space an accurate prediction of certain subset of the SUSY mass spectra requires a knowledge of the input data to an accuracy which is an order of magnitude more severe than what is currently known. Further it is found that the proximity to the Landau pole

often leads to the lightest stop mass turning tachyonic which eliminates significant regions of the parameter space.

The outline of this paper is as follows: In Sec.2, we give a brief review of the Landau fixed point analysis for the top quark Yukawa coupling. We then discuss two cases in which one can approach the Landau pole. These cases correspond to fixing the input data (such as the value of the soft SUSY breaking trilinear coupling) either at the GUT scale or at the electroweak scale. In Sec.3, we discuss the magnification of errors that can occur when one is close to the Landau fixed point. The effect of these magnifications on precision calculations at the electroweak scale is discussed. We also discuss in this section the breakdown of perturbation theory as we approach the Landau pole due appearance of $\log(\epsilon)/\epsilon$ terms (where ϵ measures the nearness to the Landau pole). In Sec. 4, we discuss the condition that the scalar sector of the SUSY mass spectra not develop tachyonic modes. We then use this constraint to determine the allowed range of A_t . Conclusions are given in Sec. 5.

2. The Top Quark Landau Pole

Since the radiative breaking of the electroweak symmetry involves a large extrapolation, from the grand unification scale M_G to the electroweak scale M_{ew} , the stability of solutions to small corrections needs to be addressed. A number of possible corrections from various sources have been discussed in the literature. These include two and higher loop corrections to beta functions, threshold corrections at the GUT scale due to heavy particles resulting from the breaking of the GUT group, threshold corrections at the electroweak scale due to the light SUSY spectrum, loop corrections to the effective potential and corrections due to possible higher dimensional operators from quantum gravity effects. Some of these issues have been studied in [14,15]. All of these corrections have influence on precision calculations at the electroweak scale. In addition, there exist another type of corrections that may be magnified due to proximity to the quasi fixed point of the top quark Yukawa Coupling [16-20]. It is well known that the one-loop RG equation analysis using the gauge and Yukawa coupling evolution below M_G gives rise to a quasi-infrared

fixed point for the top quark Yukawa coupling corresponding to $D_0 = 0$ where [16]:

$$Y_0 = \frac{Y_t}{E(t)D_0}, \quad (2.1)$$

with

$$D_0 = 1 - 6Y_t \frac{F(t)}{E(t)}. \quad (2.2)$$

Here in equations (2.1) and (2.2), $t = 2 \ln(M_G/Q)$, $Y_t = h_t^2/(4\pi)^2$, h_t is the top quark Yukawa coupling constant at the electroweak scale, Y_0 is the value of Y_t at the GUT scale (i.e. at $t = 0$) and Q is the running mass scale. $E(t)$ and $F(t)$ are two form factors defined in [16] which only depend on the gauge coupling RG equations:

$$F(t) = \int_0^t E(t) dt, \quad (2.3)$$

and at the one loop level, $E(t)$ maybe solved as,

$$E(t) = (1 + \beta_3)^{\frac{16}{3b_3}} (1 + \beta_2)^{\frac{3}{b_2}} (1 + \beta_1)^{\frac{13}{9b_1}}, \quad (2.4)$$

where $\beta_i = \alpha_i(0)b_i/4\pi$, b_i are the coefficients of the one loop beta functions, given by $(b_1, b_2, b_3) = (33/5, 1, -3)$, $\alpha_i(0)$ are the gauge coupling constant at the GUT scale, $\alpha_i(0) = \alpha_G$ and $\alpha_1 = (5/3)\alpha_Y$ with Y being the hypercharge in the Standard Model. From equation (2.2), one observes that a fixed point in the top quark Yukawa coupling corresponds to $Y_t^f(t) = E(t)/6F(t)$. The existence of this fixed point implies a fixed point mass for the top quark:

$$m_t^f = (8\pi/\alpha_2(t))^{1/2} (Y_t^f(t))^{1/2} M_Z \cos \theta_w \sin \beta, \quad (2.5)$$

where $\sin^2 \theta_w = 0.2328$ is the weak mixing angle. Thus, for $M_G = 10^{16.187}$ GeV, $\alpha_G = 1/24.0$, and $M_{ew} = M_Z$, one has $E(t) = 12.906$, $F(t) = 263.954$, and hence $Y_t^f(t) = 8.149 \times 10^{-3}$ and

$$m_t^f = 2.163 M_Z \sin \beta = 197.25 \sin \beta. \quad (2.6)$$

Because of the coupled nature of the RG equations, the same fixed point appears in some of the other parameters of the theory. Near the fixed point small variations in some of the

input quantities can be magnified in the output. Specifically, we shall show in the next section that the computed quantities in certain regions of the parameter space can be very sensitive to variations in α_G , m_t and $\tan\beta$ near a fixed point.

As is evident from the discussion above all these sensitivities in the parameters can be traced back to the fact that in the parameter space one is very close to the top quark Landau pole, which is determined by D_0 being close to zero. We note that D_0 depends on the gauge coupling constant RG equations only through a combination $E(t)/6F(t)$ and on Y_t at the electroweak scale. It is thus important to investigate the influence on this quantity from variations of the input parameters such as the grand unification scale and the GUT coupling constants etc. We consider first the size of two loop effects on the computation of the SUSY mass spectrum. In the following we discuss the relative size of the two loop effects on α 's relative to variations in the value of α_G . The two loop RG equations for the gauge couplings are

$$\frac{d\tilde{\alpha}_i}{dt} = -[b_i + \sum_j b_{ij}\tilde{\alpha}_j - a_{it}Y_t]\tilde{\alpha}_i^2, \quad (2.7)$$

where $\tilde{\alpha}_i(t) = \alpha_i(t)/4\pi$, for $i, j = 1, 2, 3$. In our analysis we have used the following values for α_i at the electroweak scale,

$$\alpha_1 = 0.016985 \pm 0.000020, \quad \alpha_2 = 0.03358 \pm 0.000011, \quad \alpha_3 = 0.118 \pm 0.007. \quad (2.8)$$

Table 1 displays a fit to the three gauge coupling constants using $M_G = 10^{16.187}$ GeV and various values for α_G . One concludes that $\alpha_G = 24.1$ gives the best fit to the numerical values for the three gauge coupling constants given in equation (2.8), both at the one loop level and at the two loop level. One now infers from Table 1 that the variation in the GUT coupling has much larger effects on the unification than the two loop RG effects! We can similarly analyse the two loop RG effects on the mass spectrum and specifically on the top quark Landau pole. Table 2 shows a comparison between the one loop and two loop evaluations for the form factors $E(t)$, $F(t)$ and their ratio. It is seen in Table 2 that, although the one loop and two loop calculations for $E(t)$ and $F(t)$ differ by several percent, their ratio, and thus the fixed point for the top quark Yukawa coupling constant, Y_t^f , differ only by less than a percent. Again, we observe that the changes in these quantities due

to changes in α_G are much larger than the changes between the one loop and two loop calculations.

Next we discuss the question of what quantities may become singular as we approach the Landau pole. This question depends critically on what is chosen as the input and what as the output. For illustrative purposes we shall consider two cases. In each of these cases we shall use m_0 , $m_{1/2}$, $\tan\beta$, α_G , M_G , and m_t as input. However, in Case I we shall assume that the trilinear coupling A_0 is specified at the GUT scale, while in Case II we shall assume that the trilinear coupling A_t is specified at the electroweak scale. We discuss each of these cases in some detail. We consider Case I first. Here the value of the trilinear coupling at the electroweak scale, i.e., A_t is given by

$$A_t = A_0 D_0 + (m_{1/2}/m_0)(H_2 - 6Y_t H_3/E) \quad (2.9)$$

where the form factors, H_2 , H_3 are defined in [16] and below. From the above we see that for fixed A_0 there is no pole in A_t , rather one finds that A_t is smooth as one approaches the Landau pole, and in this limit we find that A_t takes on the value

$$A_{tP} = (m_{1/2}/m_0)(H_2 - 6Y_t H_3/E) \quad (2.10)$$

We see now that A_{tP} is independent of the value of A_0 and is determined in terms of other parameters. Thus there is a reduction by one in the number of parameters in this case. Further, here none of the computed quantities, such as squark masses, etc., exhibit a pole. This case has been extensively discussed in [18]. For Case II, A_t is assumed fixed, and A_0 can be computed in terms of A_t via the relation, $A_0 = A_t/D_0$, so that

$$A_R = A_t - (m_{1/2}/m_0)(H_2 - 6H_3 Y_t/E) \quad (2.11a)$$

where

$$H_2 = (\alpha_G/4\pi)(\frac{16}{3}h_3 + 3h_2 + \frac{13}{15}h_1) \quad (2.11b)$$

and

$$h_i = t(1 + \beta_i t)^{-1}, \quad \frac{H_3}{F} = t\frac{E}{F} - 1 \quad (2.11c)$$

Thus A_0 has a pole similar to the pole in the top Yukawa coupling constant.

We examine next the behavior of the Higgs parameter μ in the limit as one approaches the Landau pole. We begin by noting that in the radiative breaking of the electroweak symmetry, μ is determined by fixing the Z boson mass M_Z via the equation

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2 \quad (2.12)$$

where

$$m_{H_1}^2 = m_0^2 + (0.3f_1 + 1.5f_2)\tilde{\alpha}_G m_{1/2}^2 \quad (2.13a)$$

$$m_{H_2}^2 = A_0 m_{1/2} f - (kA_0^2 - hm_0^2) + em_{1/2}^2 \quad (2.13b)$$

Here f_1 , f_2 , f , k , h , and e are defined in [16]. To investigate the Landau pole limit we write the functions h , k , f , e in the form shown below using the identity $D_0 D = 1$ where $D = 1 + 6 Y_t F(t)$. We have

$$h = (3D_0 - 1)/2 \quad (2.14a)$$

$$k = 3Y_t F D_0 / E \quad (2.14b)$$

$$f = -6 H_3 Y_t D_0 / E \quad (2.14c)$$

$$e = (3/2)[(G_1 D_0 + Y_t G_2 / E) + (H_2 D_0 + 6H_4 Y_t / E)^2 / 3 + H_8] \quad (2.14d)$$

From the above we find that $m_{H_1}^2$ is smooth while $m_{H_2}^2$ develops a pole, i.e., one has

$$m_{H_2}^2(pole) = -3Y_t(F/E)A_R^2/D_0 \quad (2.15)$$

From eq. (2.12) one can obtain the limit of μ^2 in the vicinity of the Landau pole to be

$$\mu^2 = \mu_R^2/D_0 + \mu^2(NP) \quad (2.16a)$$

where μ_R^2 is given by

$$\mu_R^2 = 3 \frac{\tan^2 \beta}{\tan^2 \beta - 1} Y_t \frac{F}{E} A_R^2 \quad (2.16b)$$

and where $\mu^2(NP)$ stands for the nonpole piece of μ^2 . We see from the above that the limit of quantities depends strongly on whether A_0 or A_t are assumed fixed as we approach the fixed point. For the case when A_0 is kept fixed the limit is smooth, while the case

when A is kept fixed induces a pole in several other parameters in the theory. The effect of these poles on electroweak physics is investigated in the next section.

3. Effect of the Landau Pole on Physics at the Electroweak Scale

As discussed in Sec. II for the case where A_t is fixed, several of the soft SUSY breaking parameters and the parameter μ^2 develop Landau poles. Computed quantities that depend on these parameters can also develop poles and show rapid variations as we approach the fixed point. There are several elements of the SUSY mass spectra which show such a behavior. We recall that the stop quark mass matrix takes the following form,

$$\begin{pmatrix} m_{\tilde{t}_L}^2 & m_t(A_t + \mu \cot \beta) \\ m_t(A_t + \mu \cot \beta) & m_{\tilde{t}_R}^2 \end{pmatrix}. \quad (3.1)$$

where

$$m_{\tilde{t}_L}^2 = \frac{2}{3}m_0^2 + \frac{1}{3}m_{H_2}^2 + \left(-\frac{1}{15}f_1 + f_2 + \frac{8}{3}f_3\right)\tilde{\alpha}_G m_{1/2}^2 + m_t^2 + \left(\frac{1}{2} - \frac{2}{3}\sin^2 \theta_w\right)M_Z^2 \cos 2\beta \quad (3.2)$$

$$m_{\tilde{t}_R}^2 = \frac{1}{3}m_0^2 + \frac{2}{3}m_{H_2}^2 + \left(\frac{1}{3}f_1 - f_2 + \frac{8}{3}f_3\right)\tilde{\alpha}_G m_{1/2}^2 + m_t^2 + \frac{2}{3}\sin^2 \theta_w M_Z^2 \cos 2\beta. \quad (3.3)$$

and $m_{H_1}^2$ and $m_{H_2}^2$ are as defined in eqs. (2.13). In the following analysis we shall assume that A_t at the electroweak scale is given and its value at the grand unification scale A_0 is determined by eq. (2.11).

The light stop mass is given by

$$m_{\tilde{t}_1}^2 = \frac{1}{2} \left[(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2) - \sqrt{(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)^2 + 4(A_t + \mu \cot \beta)^2 m_t^2} \right]. \quad (3.4)$$

while $m_{\tilde{t}_2}^2$ is given by a formula identical to eq. (3.4) except that the relative sign between the two terms is positive. We can decompose $m_{\tilde{t}_1}^2$ and $m_{\tilde{t}_2}^2$ into a pole and a non-pole piece. We exhibit the pole part below. For $m_{\tilde{t}_1}^2$ we have

$$m_{\tilde{t}_1}^2 = -2x/D_0 + m_{\tilde{t}_1}^2(NP) \quad (3.5)$$

where $x = Y_t A_R^2 F/E$. Similarly for $m_{\tilde{t}_2}^2$ we have

$$m_{\tilde{t}_2}^2 = -x/D_0 + m_{\tilde{t}_2}^2(NP) \quad (3.6)$$

From the above it is seen that although $m_{\tilde{t}_1}^2$ is numerically smaller than $m_{\tilde{t}_2}^2$ the pole part is larger in magnitude for $m_{\tilde{t}_1}^2$ than for $m_{\tilde{t}_2}^2$. Consequently the variation of $m_{\tilde{t}_1}^2$ is always larger than the variation of $m_{\tilde{t}_2}^2$ as we approach the quasi fixed point. Further the proximity of the fixed point makes the transition of $m_{\tilde{t}_1}^2$ to the tachyonic mode rapid and one expects large regions of parameter space to be eliminated when one is near the fixed point.

We note that the first two generations of squarks and the three generations of sleptons have in general a negligible correction due to the fact that the H_2 -Higgs coupling to their corresponding quark and lepton masses is negligible. Thus for these states one does not expect any rapid variations in the vicinity of the Landau pole. However, rapid variations occur for some of the chargino and the neutralino spectrum as we approach the Landau pole. This can be seen most easily in the scaling limit where the chargino spectrum is given by

$$m_{\tilde{W}_1} \simeq \tilde{m}_2, \quad m_{\tilde{W}_2} \simeq \mu \quad (3.7)$$

where $\tilde{m}_2 = (\alpha_2/\alpha_G)m_{1/2}$. From the above we infer that $m_{\tilde{W}_1}$ will show no variation while $m_{\tilde{W}_2}$ will show a significant variation as we approach the Landau pole. Similarly the neutralino masses in the scaling limit are given by

$$m_{\tilde{Z}_1} \simeq m_{\tilde{W}_1}/2, \quad m_{\tilde{Z}_2} \simeq m_{\tilde{W}_1} \quad (3.8a)$$

$$m_{\tilde{Z}_3} \simeq \mu, \quad m_{\tilde{Z}_4} \simeq \mu \quad (3.8b)$$

Here again as we approach the fixed point we do not expect any rapid variation in $m_{\tilde{Z}_1}$ and $m_{\tilde{Z}_2}$. However, $m_{\tilde{Z}_3}$ and $m_{\tilde{Z}_4}$ are expected to show a rapid variation in the vicinity of the Landau pole. These expectations are borne out in numerical analyses of the chargino and neutralino mass spectra. A similar analysis can be carried out for the Higgs. For the lightest Higgs the parameter μ enters only at the loop level and its dependence in the lightest Higgs mass is suppressed. Thus no rapid variation is expected for the lightest Higgs in the vicinity of the Landau pole. The three remaining Higgs in the scaling limit are all approximately degenerate with a common mass $m_A^2 \simeq 2m_3^2/\sin 2\beta$. One can determine m_3^2 from the radiative breaking equation

$$\sin 2\beta = 2m_3^2/(2\mu^2 + m_{H_1}^2 + m_{H_2}^2) \quad (3.9a)$$

Using Eqs(2.15) and (2.16) one finds

$$m_A^2 = -\frac{3}{\cos(2\beta)} Y_t \frac{F}{E} A_R^2 \frac{1}{D_0} + m_A^2(NP) \quad (3.9b)$$

We see that the three heavy Higgs possess a pole piece and thus show a rapid variation as we approach the Landau pole. These expectations are again borne out by numerical computations of the mass spectra. m_3^2 is determined by the renormalization group to be

$$m_3^2 = -B_0\mu^2 + r\mu_0 m_{1/2} + sA_0\mu_0 \quad (3.10)$$

Here $\mu^2 = \mu_0^2 q$ and r, s, q are defined in [16]. One finds then that B_0 becomes singular as we approach the Landau pole.

Next we discuss the effect on electroweak physics due to small changes in the input parameters when one is in the vicinity of the Landau pole. The input parameters for the case under discussion are $p_i = (m_0, m_{1/2}, A_t, \tan\beta, \alpha_G, M_G, m_t), i = 1, \dots, 7$. Let us denote by Q_a the output quantities which exhibit a pole structure near the fixed point:

$$Q_a = C_a/D_0 + Q_a(NP) \quad (3.11)$$

We denote by $Q_{a,i}$ the derivative of Q_a with respect to p_i . Then

$$Q_{a,i} = C_{a,i}/D_0 - C_a D_{0,i}/D_0^2 + Q_a(NP)_{,i} \quad (3.12)$$

It is now seen that the largest variations arise for those parameters for which $D_{0,i}$ are non-vanishing. There are three such parameters: m_t , α_G , and $\tan\beta$. We shall call these order two parameters since their variations involve a double pole. The effect on electroweak physics of the variations of these parameters is important since there are currently significant experimental errors in the determinations of, for example, m_t , and α_G . These experimental errors can be vastly exaggerated if one is in the vicinity of the Landau pole. We exhibit this magnification of errors in Fig1. Here the $m_{t_1}^2$ mass is plotted as a function of m_t for various assumed values of $\tan\beta$. In the example shown (for the case $A_t = 0.5$) one finds that the $m_{t_1}^2$ mass begins to show a very rapid variations as we approach the Landau pole. Thus in the vicinity of the Landau pole changes in m_t of the order of a

few GeV can lead to changes in the $m_{\tilde{t}_1}^2$ mass of several hundred GeV. Similarly a small fractional change in $\tan\beta$ can generate a similar large change in $m_{\tilde{t}_1}^2$ mass. A similar phenomenon occurs for variations in α_G . This means that precision analyses of mass spectra will require a very high degree of accuracy in the parameters m_t , α_G if one happens to lie in the vicinity of the Landau pole.

Fig 1 also shows that the stop 1 turns tachyonic as we approach the Landau pole. It can be seen from eq(3.5) that the residue of the pole term is negative and thus as m_t approaches m_t^f the pole term cancels the non-pole term in eq(3.5) turning \tilde{t}_1 tachyonic. We wish to point out the important role that the trilinear coupling A_t and its sign play in turning \tilde{t}_1 tachyonic. Actually the parameter which controls the approach to tachyonic limit is A_R which is defined by eq(2.11). In the vicinity of the pole $x \simeq A_R^2/6$. To get an idea of the sizes of various entries in eq(2.11a) we find that for the same parameters as used in eq(2.6) one has

$$A_R \simeq A_t - 0.613m_{\tilde{g}} \quad (3.13)$$

One finds then that for A_t positive there is a cancellation between the two terms on the right hand side of eq(3.13) which reduces the size of A_R and slows down the approach to the tachyonic limit. Thus for positive A_t one will have to get relatively close to the Landau pole for \tilde{t}_1 to turn tachyonic. For the A_t negative case there is a reinforcement between the two terms on the right hand side of eq(3.13). Thus in this case approach to the tachyonic limit is much faster. An illustration of these results is given in Fig 2. Here one sees clearly the strong influence that the Landau pole has in turning the stop 1 tachyonic. A more detailed investigation of the tachyonic condition is given in Sec 4, where the tachyonic limit is used to delineate the allowed parameter space of MSGM.

The coefficient of the double pole term exhibits certain scaling laws. Thus neglecting the single pole and the non-pole terms one has

$$Q_{a,i}/Q_{a,j} = D_{0,i}/D_{0,j} \quad (3.14)$$

From the above we see that the ratio of variations is independent of the index a and hence also independent of all the parameters that do not enter in D_0 . If we label the order two

variables m_t , α_G , and $\sin\beta$ by $i = 1, 2, 3$, then

$$Q_{a,1}/Q_{a,3} = -\sin\beta/m_t \quad (3.15a)$$

$$Q_{a,2}/Q_{a,1} = -(1/2)m_t \, d(\log(F/E))/d(\alpha_G) \quad (3.15b)$$

We emphasize, however, that there are important non pole corrections which one must include in any realistic evaluation of the variations.

We wish to investigate next the issue of how close one may approach the Landau pole. First from the stability of the the potential at the GUT scale one requires that

$$A_0^2 < 3(3m_0^2 + \mu_0^2) \quad (3.16)$$

Now

$$\mu^2 = \mu_0^2(1 + \beta_2 t)^{3/b_2}(1 + \beta_1 t)^{3/b_1}(D_0)^{1/2} \quad (3.17)$$

We have already seen that from radiative breaking of the electro-weak symmetry one deduces μ^2 to have the form $\mu^2 = a_1 + a_2/D_0$. Using these results we can deduce the D_0 dependence of μ_0^2 . One finds

$$\mu_0^2 = a/D_0^{1/2} + b/D_0^{3/2} \quad (3.18)$$

Using eq. (3.18) the condition of stability of the potential at the GUT scale takes the form

$$3m_0^2 D_0^2 + a D_0^{3/2} + b D_0^{1/2} - A_R^2/3 > 0 \quad (3.19)$$

If one is close enough to the pole that powers of D_0 higher than $D_0^{1/2}$ can be neglected then one obtains

$$D_0^{1/2} > A_R^2/(3b) \quad (3.20)$$

The above equation gives the point of closest approach to the Landau pole. However, as one gets very close to the fixed point other phenomenon must be taken into account. These include higher order terms in the top quark Yukawa coupling equations, loop effects in the radiative breaking equations etc. Here we point out an interesting phenomenon associated with inclusion of loop effects in radiative breaking and its implication in the

Landau pole analysis. With inclusion of one-loop effects to the effective potential, the condition of eq. (2.12) is modified as follows:

$$\mu^2 = \frac{m_{H_1}^2 + \Sigma^1 - (m_{H_2}^2 + \Sigma^2) \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2 \quad (3.21)$$

where $\Sigma^{1,2}$ are the one loop corrections. The largest contributions to $\Sigma^{1,2}$ arise from the stops and are given in the Appendix. Using results of the Appendix in eq. (3.21) we find that the satisfaction of radiative electro-weak symmetry breaking now shows that the solution of μ^2 with the above corrections is more complicated. It has the form

$$\mu^2 = a_1 + b_1/\epsilon + c_1 \log(d_1 \epsilon)/\epsilon \quad (3.22)$$

where $\epsilon = D_0$. From eq. (3.22), we see that inclusion of the one loop correction in the effective potential brings in the new feature of the $\log(\epsilon)/\epsilon$ term while the tree contribution is only $O(1/\epsilon)$. This means that as we approach the Landau pole the contribution of the one loop effective potential eventually becomes larger than the tree contribution due to the $\log(\epsilon)$ divergence. This would signal the breakdown of the perturbation theory. Thus perturbation theory puts a natural cutoff on how close one can approach the Landau pole. The point of closest approach is defined by the condition that the loop contribution in eq. (3.22) be smaller than the tree contribution.

4. The Parameter Space of the Minimal Supergravity Model

In this section we shall discuss the implications of the condition that the SUSY spectrum be free of tachyons in the analysis of the radiative electroweak symmetry breaking. As it turns out, of the 32 SUSY particles, only the light stop may become tachyonic in some region of the parameters. From eq. (3.5) it is easy to understand why the light stop can become tachyonic. Since $x > 0$, the first term of Eq. (3.5) will always dominate the second for D_0 small enough. Thus as one approaches the Landau pole, $m_{t_1}^2$ will always turn tachyonic, provided eq. (3.20) is not violated. One can then ask when the light stop becomes tachyonic. The answer is very sensitive to the values one chooses for the parameters, since c , e , f , f_1 , f_2 , f_3 , h , and k have very complicated dependence on M_G , α_G , m_t (via Y_t), and $\tan \beta$. To determine if the light stop is tachyonic, we need to solve the

equation $m_{\tilde{t}_1}^2 = 0$. Fortunately, this equation can be solved analytically for $A_t = 0$. We consider two points in the parameter space for $A_t = 0$, $\tan \beta = 1.732$, and $m_t = 150$ GeV.

Case 1): We choose the remaining parameters to be $M_G = 10^{16.1}$ GeV, $\alpha_G = 1/25.7$ (we may replace $M_Z^2 \cos^2 \theta_W$ by M_W^2 while computing Y_t at the electroweak scale), and $\alpha_2 = 0.03322$. Scaling all the masses by 100 GeV, the light stop has zero mass on the $m_{1/2}$ and m_0 plane along the following line:

$$19.66 - 15.11m_{1/2}^2 - 4.54m_{1/2}^4 + 3.77m_0^2 + 0.74m_{1/2}^2m_0^2 + 0.39m_0^4 = 0, \quad (4.1)$$

Case 2): We choose remaining parameters to be $M_G = 10^{16.187}$ GeV, $\alpha_G = 1/24.11$, and $\alpha_2 = 0.03358$. Using the same scaling as in case 1, the light stop has zero mass on the $m_{1/2}$ and m_0 plane along the following line:

$$19.04 + 28.44m_{1/2}^2 + 18.41m_{1/2}^4 + 6.16m_0^2 + 6.47m_{1/2}^2m_0^2 + 0.55m_0^4 = 0, \quad (4.2)$$

There is a significant difference between Case 1 and Case 2. For Case 1, coefficients in eq (4.1) appear with both positive and negative signs, while in Case 2, coefficients in eq (4.2) all have positive signs. Thus, the condition that the light stop be tachyonic may be satisfied in Case 1 for points below the line given by equation (4.1), while this same condition cannot be satisfied at any point on the m_0 - $m_{1/2}$ plane for Case 2, even though the two cases differ by only a small change in α_G and other parameters. Cases 1 and 2 discussed above were for illustrative purposes only. We discuss next a more systematic study of the condition that the light stop be nontachyonic. Figures 3 and 4 show the results for $m_t = 170$ GeV (corresponding to a pole mass of about 175 GeV), and $\tan \beta = 5.0$. Fig. 3 shows the results for $A_t/m_0 = -0.25, -0.3, -0.4, -0.5$, and -0.8 on the $m_{\tilde{g}}$ - m_0 plane, the light stop is tachyonic above and right to each curve for a fixed A_t . Fig. 4 demonstrates the results for $A_t/m_0 = 0.5, 0.8, 1.0, 2.0$, and 5.0 on the $m_{\tilde{g}}$ - m_0 plane, the light stop is tachyonic above and left to each curve for a fixed A_t . From these figures, one observes that the condition that the light stop be nontachyonic excludes more parameter space when μ and A_t have the same sign than when they have opposite signs. Further large values of A_t of either sign, specifically values of $A_t < -0.8m_0$ and $A_t > 5.0m_0$, are excluded by this condition. Thus the allowed region of A_t lies within $-0.8m_0 < A_t < 5.0m_0$ for

$\tan\beta=5$ and $m_t=170$ GeV. This is a very strong constraint and it is interesting that the light stop being tachyonic alone can impose such a stringent constraint on the parameter space of MSGM.

5. Conclusions

We have investigated in this paper the phenomena associated with effects of the Landau pole in the top quark Yukawa coupling and implications of the constraint that the SUSY spectrum be free of tachyons. Regarding the first we find that in the renormalization group analyses some of the mass spectra (i.e., masses of the stops, the heavy chargino, heavy neutralinos, and the heavy Higgs) are very sensitive to the input errors in m_t , α_G , and $\tan\beta$ when the top mass is close to its fixed point value. In this region of the parameter space precision physics requires much greater accuracy of the input data. We also investigated the issue of how close one may approach the fixed point. Regarding the second phenomenon the condition that the SUSY spectrum be tachyon free turns out to be a rather stringent one as it eliminates a large part of the parameter space. For example, the trilinear scalar coupling, i.e., A_t is confined within the range of $-0.8m_0$ to $5.0m_0$, for the parameters of Fig. 3 and 4. The graphs indicate the strong correlation between A_t and μ , i.e., the light stop is more likely to become tachyonic when A_t and μ have the same sign.

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Appendix

The contribution to Σ^1 from stops $\tilde{t}_{1,2}$ is given by

$$\begin{aligned} \Sigma_{\tilde{t}_{1,2}}^2 = & 3 \frac{\alpha_2 + \alpha_Y}{8\pi} m_{\tilde{t}_{1,2}}^2 \left[\frac{1}{4} \mp \frac{1}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} \left\{ \frac{1}{2} (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2) \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right) \right. \right. \\ & \left. \left. + \frac{m_t^2}{M_Z^2 \sin^2 \beta} \mu (\mu + \tan\beta A_t) \right\} \right] \log \left(\frac{m_{\tilde{t}_{1,2}}^2}{eQ^2} \right). \end{aligned} \quad (A.1)$$

Similarly the contribution to Σ^2 from stops $\tilde{t}_{1,2}$ is

$$\begin{aligned}\Sigma_{\tilde{t}_{1,2}}^2 = & 3 \frac{\alpha_2 + \alpha_Y}{8\pi} m_{\tilde{t}_{1,2}}^2 \left[\frac{-1}{4} + \frac{m_t^2}{M_Z^2 \sin^2 \beta} \mp \frac{1}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} \left\{ \frac{1}{2} (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2) \left(-\frac{1}{2} + \frac{4}{3} \sin^2 \theta_W \right) \right. \right. \\ & \left. \left. + \frac{m_t^2}{M_Z^2 \sin^2 \beta} m_0 A_t (\mu \cot \beta + m_0 A_t) \right\} \right] \log \left(\frac{m_{\tilde{t}_{1,2}}^2}{eQ^2} \right).\end{aligned}\quad (A.2)$$

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Table Captions

Table 1 The fit to the electroweak data for the gauge coupling constants given in equation (2.9) in one and two loop approximations, for $M_G = 10^{16.187}$ GeV, and various values for α_G .

Table 2 The effects of the variation of α_{GUT} on the ratio F/E and hence on the top

Landau pole in one and two loop R.G.analysis. One finds that the variations of α_{GUT} are much more significant than the two loops effects.

Figure Captions

Fig. 1 Exhibition of $m_{\tilde{t}_1}^2$ vs. m_t for the case when $A_t = 0.5m_0$, $m_0 = 600$ GeV, $m_{\tilde{g}} = 300$ GeV and $\tan\beta = 1.2, 1.4, 2.0, 3.0, 5.0, 7.0, 9.0$ in increasing value as we go left to right. We see that there is a huge magnification of error in $m_{\tilde{t}_1}^2$ for a corresponding small error in m_t when the top mass is close to its fixed point value.

Fig. 2 Exhibition of the effect of the Landau pole on the tachyonic limit for the case $m_0=600$ GeV, $m_{gluino}=300$ GeV and $\mu < 0$. The top curve gives the position of the Landau pole as a function of $\tan\beta$. The middle curve gives, for the case when $A_t = 0.5m_0$, the value of m_t as a function of $\tan\beta$ so that for all m_t values above this value $m_{\tilde{t}_1}^2$ is always tachyonic. The bottom curve is the same as the middle curve except that $A_t = -0.5m_0$.

Fig. 3 The solutions for $m_{\tilde{t}_1}^2 = 0$ on the $m_{\tilde{g}}-m_0$ plane for $\alpha_G = 24.11$, $M_G = 10^{16.187}$ GeV, and $m_t = 170$ GeV and $\tan\beta = 5.0$. This figure shows the branch for $A_t > 0$. The light stop is tachyonic above and left to each curve for a fixed A_t . All masses are in GeV.

Fig. 4 The solutions for $m_{\tilde{t}_1}^2 = 0$ on the $m_{\tilde{g}}-m_0$ plane for $\alpha_G = 24.11$, $M_G = 10^{16.187}$ GeV, and $m_t = 170$ GeV and $\tan\beta = 5.0$. This figure shows the branch for $A_t < 0$. The light stop is tachyonic above and right to each curve for a fixed A_t . All masses are in GeV.

Table 1

	One Loop	Two Loop
$\alpha_G^{-1} = 25.7$		
α_1	0.0166359	0.0165156
α_2	0.0323481	0.031735
α_3	0.0994	
$\alpha_G^{-1} = 24.5$		
α_1	0.0169748	0.0168391
α_2	0.0336544	0.0329366
α_3	0.1129	
$\alpha_G^{-1} = 24.1$		
α_1	0.0170909	0.0169493
α_2	0.0341137	0.0333552
α_3	0.1182	
$\alpha_G^{-1} = 24.0$		
α_1	0.0171201	0.016977
α_2	0.0342304	0.0334613
α_3	0.1196	

Table 2

	One Loop	Two Loop
$\alpha_G^{-1} = 24.11$		
E	12.985	13.290
F	266.48	275.97
F/E	20.523	20.766
$\alpha_G^{-1} = 24.5$		
E	11.938	12.536
F	252.61	266.78
F/E	21.160	21.288
$\alpha_G^{-1} = 25.0$		
E	11.101	12.536
F	242.46	256.15
F/E	21.842	21.919